ECOM 111

The continuity equation and minority carrier diffusion equation (Problems)

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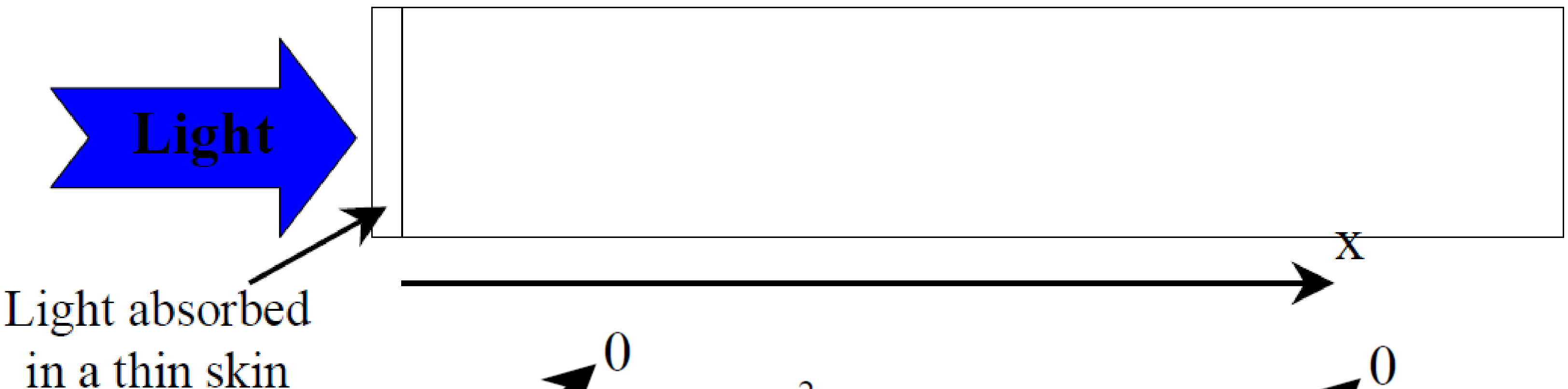
Example 1

Consider a **semi-infinite** p-type silicon sample with $N_A=10^{15}$ cm^{-3} **constantly** illuminated by light absorbed in a **very thin region** of the material creating a steady state excess of 10^{13} cm^{-3} minority carriers. What is the minority carrier distribution in the region x>0?



$$\cancelto{0}{\frac{\partial(\Delta n_p)}{\partial t}} = D_N \frac{\partial^2(\Delta n_p)}{\partial x^2} - \frac{(\Delta n_p)}{\tau_n} + \cancelto{0}{G_L}$$

$$D_N \frac{\partial^2(\Delta n_p)}{\partial x^2} = \frac{(\Delta n_p)}{\tau_n}$$

General Solution …

$$\Delta n_p(x) = Ae^{(-x/L_N)} + Be^{(+x/L_N)} \quad where \quad L_N \equiv \sqrt{D_n \tau_n}$$

L_N is the "diffusion length" the average distance a minority carrier can move before recombining with a majority carrier.

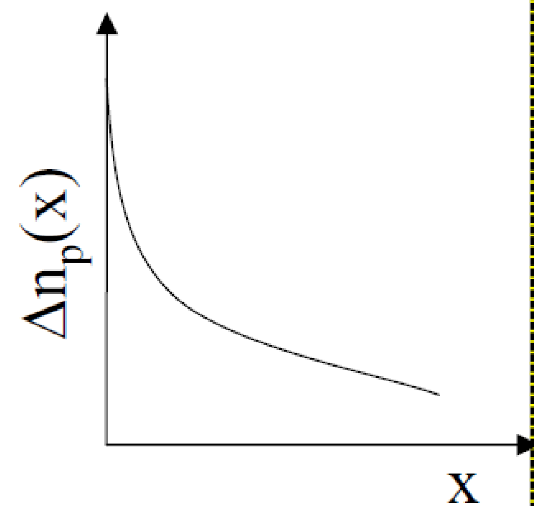
Boundary Condition …

$$\Delta n_p(x=0) = 10^{13}\ cm^{-3} = A + B$$

$$\Delta n_p(x=\infty) = 0 = A(0) + Be^{(+\infty/L_N)}$$

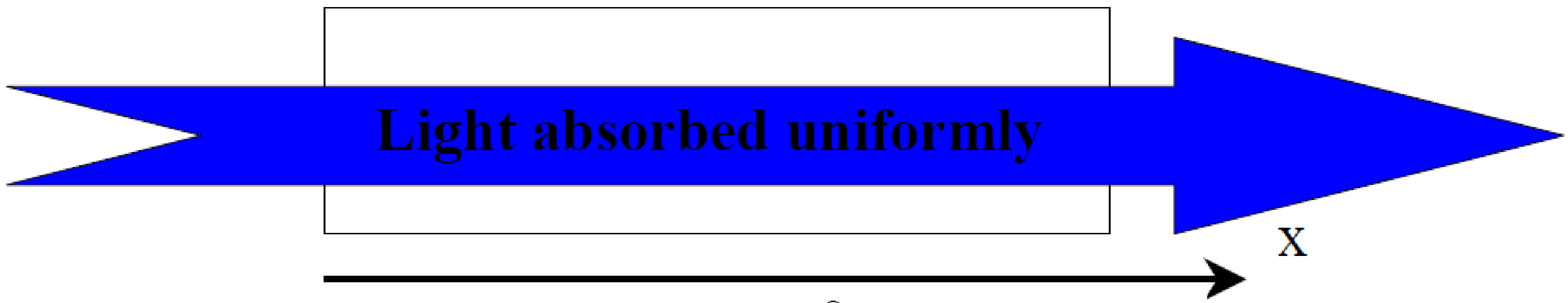
$$\Rightarrow B = 0$$

$$\boxed{\Delta n_p(x) = 10^{13}\ e^{(-x/L_N)} \quad cm^{-3}}$$



Example 2

Consider a p-type silicon sample with $N_A=10^{15}\ cm^{-3}$ and minority carrier Lifetime $\tau=10\ uS$ **constantly illuminated** **by light absorbed uniformly throughout the material creating an excess $10^{13}\ cm^{-3}$ minority carriers per second**. The light has been on for a very long time. At time t=0, the light is shut off. What is the minority carrier distribution in **for t<0?**



$$\cancelto{0}{\frac{\partial(\Delta n_p)}{\partial t}} = D_N \cancelto{0}{\frac{\partial^2(\Delta n_p)}{\partial x^2}} - \frac{(\Delta n_p)}{\tau_n} + G_L$$

$$\Delta n_p(all\ x, t<0) = G_L \tau_n = 10^7\ cm^{-3}$$

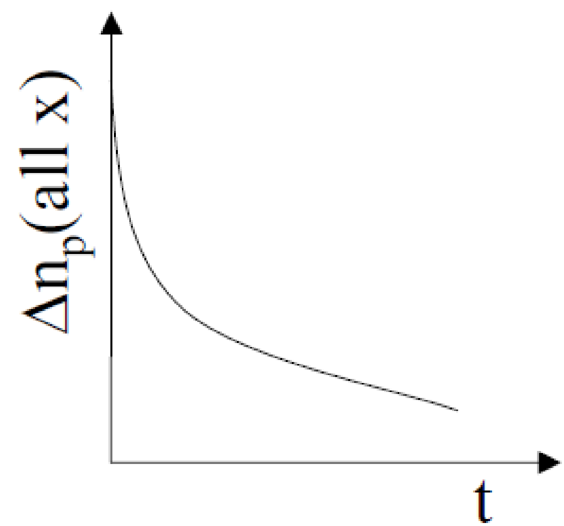
Example 3

In the previous example: What is the minority carrier distribution in for t>0?

$$\frac{\partial(\Delta n_p)}{\partial t} = D_N \cancelto{0}{\frac{\partial^2(\Delta n_p)}{\partial x^2}} - \frac{(\Delta n_p)}{\tau_n} + \cancelto{0}{G_L}$$

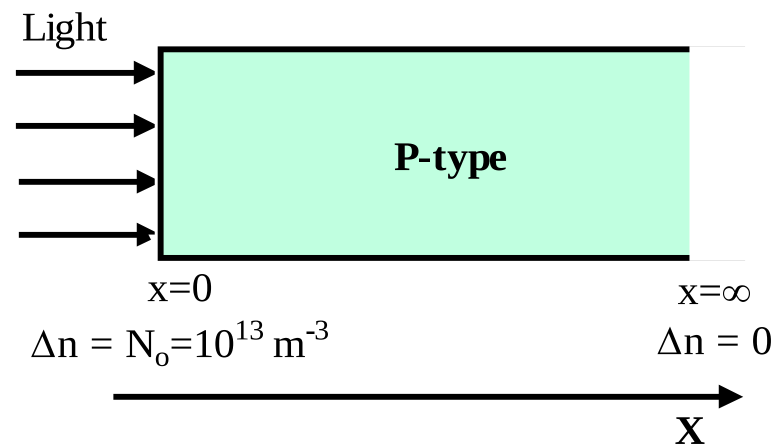
$$\Delta n_p(t) = \left[\Delta n_p(t=0)\right] e^{\left(-t/\tau_n\right)}$$

$$\boxed{\Delta n_p(t) = 10^7 e^{\left(-t/1e-5\right)}}$$



Example 4

An excess electron concentration of 10^{13} m^{-3} is **maintained** at one side of a bar of p-type semiconductor. The excess electron distribution along the bar is given by the solution of the **diffusion equation**. Calculate the diffusion current density at the side where the injection is maintained. Assume that the temperature is 300°K, the minority carriers mobility = 0.1 m^2/V.s and the minority carrier lifetime = 0.01 µs.



The minority carrier diffusion equation is given by:

$$\frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{L_e^2} = 0$$

Note: the light is absorbed in a very thin thickness in the bar so an excess of electron concentration is maintained at this side of the bar. **The entire bar does not exposed to light.**

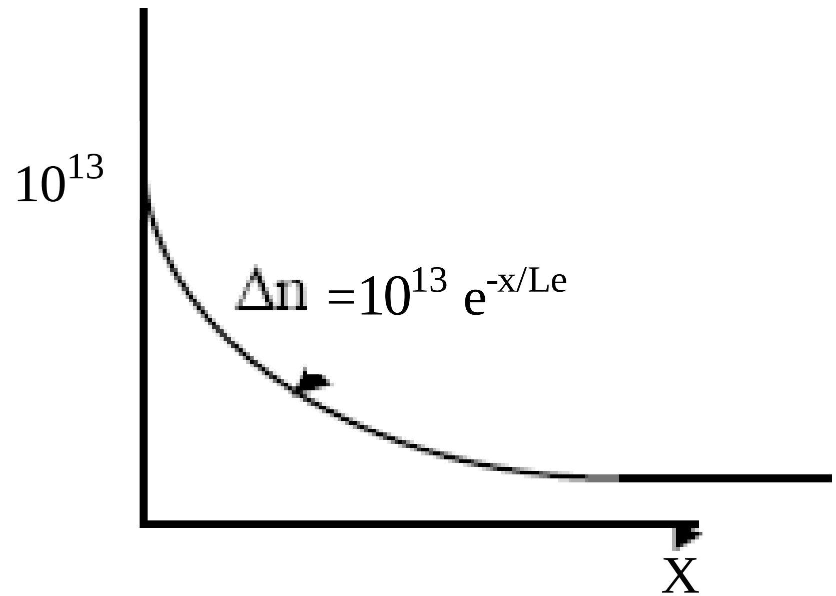
The solution of the diffusion equation is given by:

$$\Delta n(x) = C_1 e^{-\frac{x}{Le}} + C_2\, e^{\frac{x}{Le}}$$

At $x = 0$ $\quad \Delta n = N_o = 10^{13}$ $\quad\Rightarrow\quad C_1 + C_2 = 10^{13}$

At $x = \infty$ $\quad \Delta n = 0$ $\quad\Rightarrow\quad C_2 = 0$

$$\Delta n(x) = 10^{13} e^{-\frac{x}{Le}}$$



$$J_{diff} = q\, D_e \frac{d\,\Delta n}{dx}$$

$$J_{diff} = -qD_e \frac{N_o}{L_e} e^{-\frac{x}{Le}}$$

For D_e we can use eintisen relation

$$\frac{D_e}{\mu_e} = \frac{KT}{q}$$

$$D_e = 0.026 \times 0.1$$

$$D_e = 2.6 \times 10^{-3} \ \mathrm{m}^2 s^{-1}$$

For $\mathrm{L_e}$

$$L_e = \sqrt{D_e \tau_e}$$

$$L_e = \sqrt{2.6 \times 10^{-3} \times 0.01 \times 10^{-6}}$$

$$L_e = 5.1 \ \mu\mathrm{m}$$

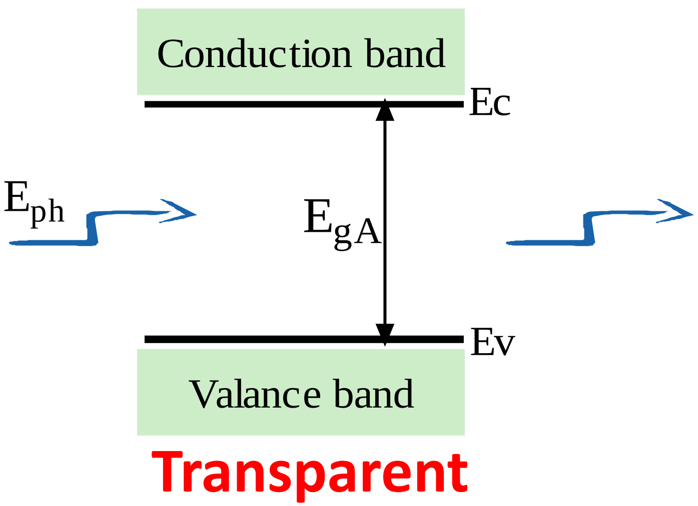
$$J_{diff} = -\frac{1.6 \times 10^{-19} \times 2.6 \times 10^{-3} \times 10^{13}}{5.1 \times 10^{-6}} = -0.815 \times 10^{-3} \ \mathrm{A/m^2}$$

The negative sign means that the direction of current flow in –ve x direction.

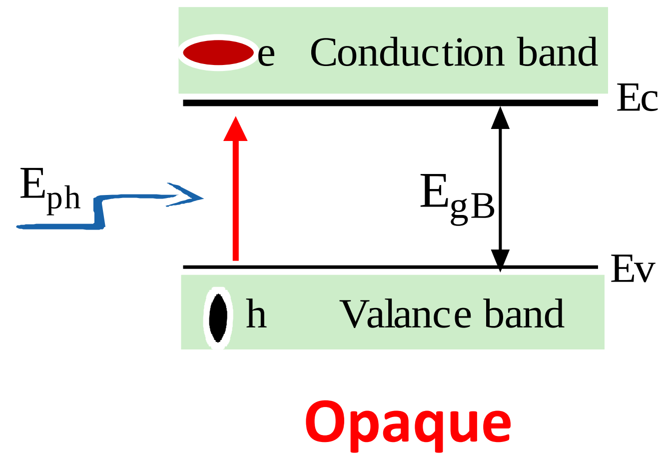
$$\left| J_{diff} \right| = 0.815 \times 10^{-3} \ \mathrm{A/m^2}$$

Example 5

a) A semiconductor "A" is transparent, while another semiconductor "B" is opaque. Which semiconductor has the larger energy gap? Why?



$$E_{ph} = \frac{ch}{\lambda_{ph}}$$



E_{ph} = photon energy J(eV)

C = light speed $= 3\times10^{8}$ m/s

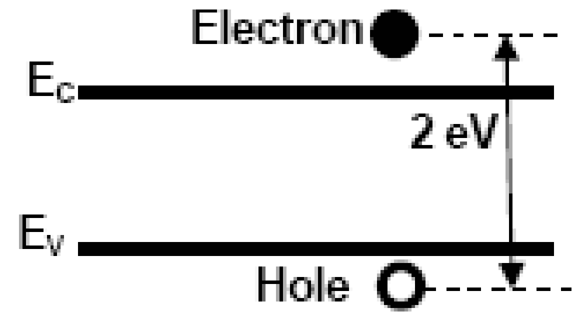
h = Blank's constant $= 6.62\times10^{-34}\,J.s$

λ_{ph} = photon wavelength m(μm)

$$E_{ph}(eV) = \frac{1.24}{\lambda_{ph}(\mu m)}$$

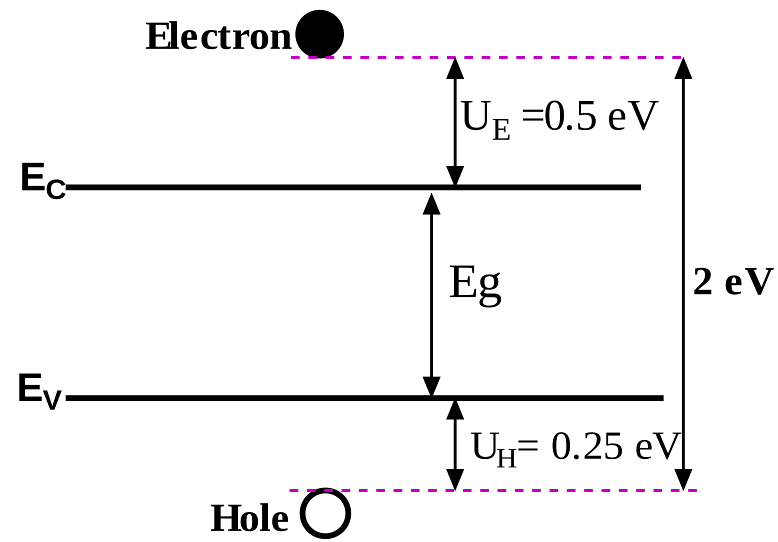
Example 6

The figure has shown the energy band structure of a semiconductor. The electron's energy is 0.5 eV and the hole's energy is 0.25 eV. Calculate the photon wavelength needed to just raise an electron from the valence band to the conduction band.



Electron energy $U_E = 0.5$ eV
Hole energy $U_H = 0.25$ eV

$U_E + U_H + E_g = 2$ eV
$E_g = 2 - (0.5 + 0.25) = 1.25$ eV



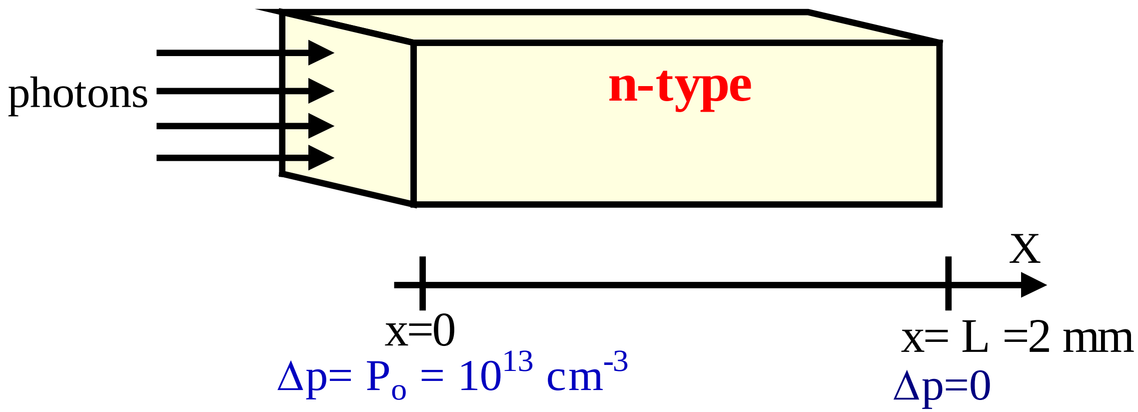
The photon wavelength needed to just raise electron from the valence band to the conduction band can be calculated using the energy gab using the following relation:

$$\lambda(\mu m) = \frac{1.24}{E_g(eV)}$$

$$\lambda = \frac{1.24}{1.25} = 0.992 \ \ \mu\text{m}$$

Example 7

At $x = 0$ photons are incident on an N-type bar of length 2 mm and hence 10^{13} cm^{-3} excess electrons and holes are generated as shown in figure below, the diffusion length for minority holes is 1 mm, and all the excess holes vanish at the other end of the bar calculate the distance x_o at which **steady state excess** minority holes drops to half its peak value.



Note:
The entire bar does not exposed to light $\mathbf{G_i = 0}$.
Since the light is absorbed in a very thin thickness in the bar so an excess of holes concentration P_o is maintained at $x = 0$

Minority carrier diffusion equation: (at steady state)

$$\frac{d^2 \Delta p(x)}{dx^2} - \frac{\Delta p(x)}{L_h^2} = 0$$

$$\Delta p(x) = C_1 \, e^{-\frac{x}{L_h}} + C_2 \, e^{\frac{x}{L_h}}$$

To find the const C_1, C_2

At $x = 0$ $\quad \Delta P = P_o = 10^{13}$ $\longrightarrow$ $C_1 + C_2 = 10^{13}$ **I**

At $x = L$ $\quad \Delta P = 0$ $\longrightarrow$ $C_1 e^{-\frac{L}{L_h}} + C_2 e^{\frac{L}{L_h}} = 0$ **II**

By solving equations I&II we get :

$C_2 = -1.865 \times 10^{11}$	$C_1 = 1.02 \times 10^{13}$

At $x = x_o$ $\Delta P = 1/2$ peak value $= 1/2\ P_o$

$$\frac{P_o}{2} = C_1\, e^{-\frac{X_o}{L_h}} + C_2\, e^{\frac{X_o}{L_h}} \qquad (\text{where } L_h = 1mm)$$

$$\frac{P_o}{2} = C_1\, e^{-X_o} + C_2\, e^{X_o} \qquad (x_o \text{ in mm}) \qquad \text{III}$$

Multiply equation III by e^{X_o}

$$\frac{P_o}{2}\, e^{X_o} = C_1 + C_2\, e^{2X_o} \qquad (\text{Arranging})$$

$$C_2\, e^{2X_o} - \frac{P_o}{2}\, e^{X_o} + C_1 = 0$$

Let $e^{X_o} = y$

$$C_2 y^2 - \frac{P_o}{2} y + C_1 = 0$$

$$-1.865\times10^{11}y^{2}-0.5\times10^{13}y+1.02\times10^{13}=0$$

$$1.865\times10^{11}y^{2}+0.5\times10^{13}y-1.02\times10^{13}=0$$

$$y=\frac{-0.5\times10^{13}\pm\sqrt{\left(0.5\times10^{13}\right)^{2}+4\times1.865\times10^{11}\times1.02\times10^{13}}}{2\times1.865\times10^{11}}$$

$$y=\frac{-0.5\times10^{13}\pm5.71\times10^{12}}{2\times1.865\times10^{11}}$$

$y = 1.904$

$e^{X_o} = 1.904$

$X_o = 0.6439$ mm

Or

$y = -28.71$

(ignored)

sience $y = e^{X_o} = -28.71$